

Totally asymmetric simple exclusion process with one or two shortcuts

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In this paper, the operation of totally asymmetric simple exclusion process with one or two shortcuts under open boundary conditions is discussed. Using both mathematical analysis and numerical simulations, we have found that, according to the method chosen by the particle at the bifurcation, the model can be separated into two different situations which lead to different results. The results obtained in this paper would be very useful in the road building, especially at the bifurcation of the road.

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I. INTRODUCTION

One-dimensional totally asymmetric simple exclusion process (TASEP) has been studied for many years [1, 2]. It is one special type of ASEP which represent one of the basic models studying non-equilibrium behavior of particle transport along one-dimensional lattices. ASEP was first introduced in [3] for the explanation of ribosome

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inside the cell of creature. Now, ASEP has also been used to simulate a lot of physical processes including surface diffusion [4], traffic model [5], and molecular motors [6],[7], etc.

TASEP, as a special case of ASEP, has two identical characters. One is that the model is discrete, which means that it is studied in finite time intervals. The other is that the particles in the lattices can only move in one direction. The analytical solutions under open boundary conditions had been obtained in [8, 9].

Under open boundary conditions, the solutions yield phase diagrams with three phases [8, 9]. At small values of injection rates $\alpha < 0.5$ and $\alpha < \beta$, the system is found in a low-density entry-limited phase where

$$\rho_1 = \alpha \quad \rho_L = \frac{\alpha(1-\alpha)}{\beta} \quad J = \alpha(1-\alpha) \quad \rho_{bulk} = \alpha \quad (1)$$

where ρ_1, ρ_L and ρ_{bulk} are the densities at the entrance, exit and the bulk of the lattice far away from the boundaries, respectively. J denotes the flux.

At small values of extraction rates $\beta < 0.5$ and $\beta < \alpha$, the system is in a high-density exit-limited phase with

$$\rho_1 = 1 - \frac{\beta(1-\beta)}{\alpha} \quad \rho_L = 1 - \beta \quad J = \beta(1-\beta) \quad \rho_{bulk} = 1 - \beta \quad (2)$$

At large values of the injection ($\alpha \leq 0.5$) and extraction ($\beta \leq 0.5$) rates the system is in a maximal current phase with

$$\rho_1 = 1 - \frac{1}{4\alpha} \quad \rho_L = \frac{1}{4\beta} \quad J = \frac{1}{4} \quad \rho_{bulk} = \frac{1}{2} \quad (3)$$

A large number of varieties of extensions of TASEPs have been investigated, such as TASEP with hierarchical long-range connections [10], two lane situations [11], two speed TASEP[12], the effect of defect locations[13], particle-dependent hopping rates [14], and so forth.

Recently, J. Brankov [15] and E. Pronina [16] studied an ASEP with two chains in the middle of the filament. They supposed that a particle chooses to move into these two chains with equal probability, 0.5 and these two chains have the same

length. Then in 2007, Yao-Ming Yuan and Rui Jiang [17] investigated a TASEP with a shortcut in the middle. They set a possibility q for a particle to jump through the shortcut when it faces the bifurcation, and they set the length of the shortcut to be zero. Unfortunately, they did not state the situation clearly.

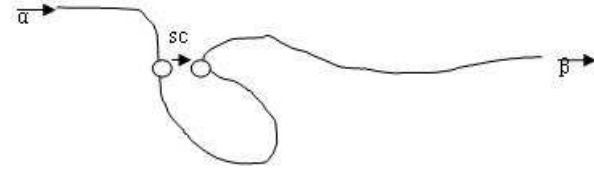
In this paper, we will investigate TASEP with one or two shortcuts respectively. The length of the shortcut is also assumed to be zero. For example, the filament on which the motor moves may be twisted as figure 1(a), a motor may have a chance to jump directly from site k_1 to k_2 , as shown in figure 1(b).

To the basic model, i.e. there is only one shortcut along the filament. It is natural for us to divide the whole filament into three segments. As shown in figure 1(b), a molecular motor will face a choice of whether to jump through the shortcut or to move ordinarily through segment 2, when it reaches site k_1 . An important problem is that the particle at site k_1 may have to wait to go through the shortcut if there is also another particle occupying site $k_2 - 1$. Which of them to go first should be determined clearly.

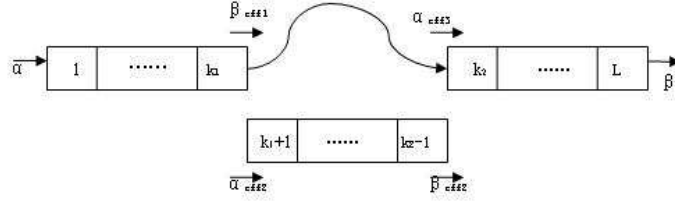
After the basic model investigated, we would like to do some research work on our advanced model 1. In advanced model 1, there are two shortcuts which begin and end at different sites. These two shortcuts stay respectively along the filament, that is to say, shortcut 2 begins after shortcut 1 ends, as shown in figure 2.

Also, considering the difference of places where the shortcuts stay, we have advanced model 2, as shown in figure 3. In advanced model 2 there are also two shortcuts. But they begin at the same site and end at different, which means when a motor moves into site k_1 , it will choose whether to pass through shortcut 1 or shortcut 2 or to go ordinarily through segment 2.

In the following, we firstly give the detailed description of the different models, then we will discuss their phase situation of the corresponding segments theoretically, which is followed by the numerical simulations.

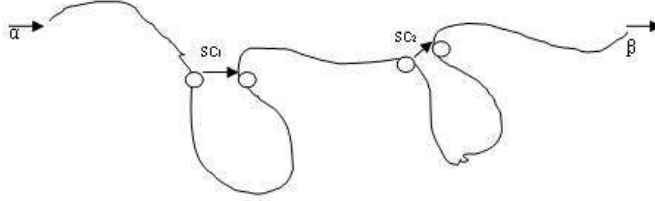


(a)

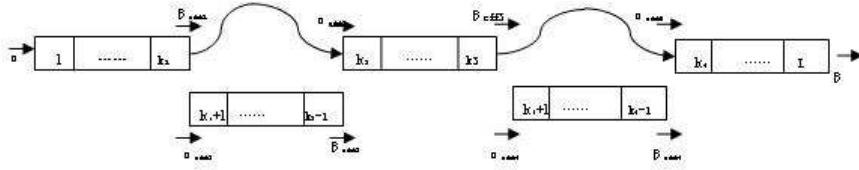


(b)

FIG. 1: basic model with one shortcut: (a)a road with one shortcut at the bulk; (b)lattices with one shortcut.



(a)



(b)

FIG. 2: advanced model with two shortcuts at different beginnings and endings: (a)a road with two shortcuts at different beginnings and endings; (b)lattices with two shortcuts at different beginnings and endings.

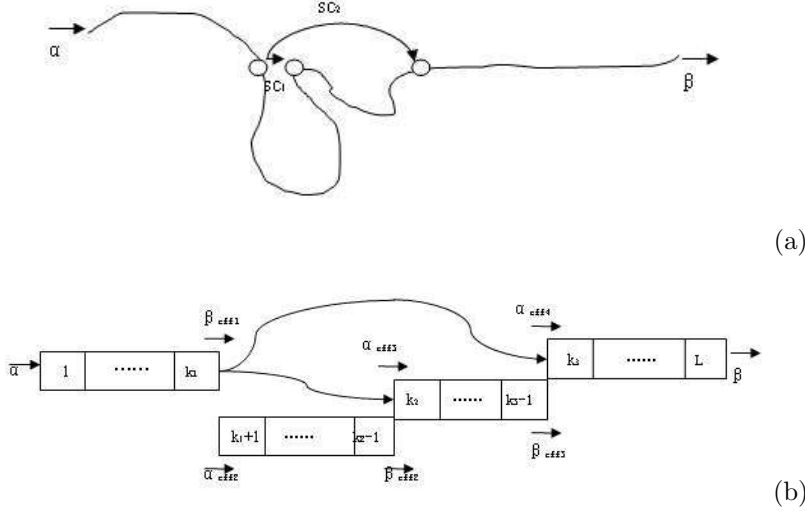


FIG. 3: advanced model with two shortcuts at the same beginning and different endings: (a) a road with two shortcuts at the same beginning and different endings; (b) lattices with two shortcuts at the same beginning and different endings.

II. MODELS

A. Basic model

First, we divide the lattices into three segments, as shown in figure 1(b). Segment 1,2,3 start from site 1, $k_1 + 1, k_2$ and end in site $k_1, k_2 - 1, L$, respectively.

In this model, all the particles traveling in the lattices are identical to each other. At time $t = 0$, we may suppose that the lattices are empty. As time goes, there will be a particle in site 1. This particle will pass continuously through the whole filament.

Given a site i ($1 \leq i \leq L$), in an infinitesimal time interval dt , if $i = 1$, a particle is inserted with probability αdt , provided the site is empty, if $i = L$ and there is a particle in it, then the particle in site L is extracted with probability βdt , and if $1 \leq i \leq L$ ($i \neq k_1, k_2 - 1$) and site i is occupied, the particle in site i will move into site $i + 1$ with probability dt , providing site $i + 1$ is empty. However, if a particle is in site k_1 or $k_2 - 1$, we have to clarify the way it moves. Although Yao-Ming Yuan, Rui Jiang [17] have discussed the method of choice at site k_1 , the situation at site $k_2 - 1$

has not been studied clearly. It is possible that when both site k_1 and site $k_2 - 1$ have particles, it may produce a collision in site k_2 .

1. Situation 1

If site k_1 has a particle, then the particle will choose the way as following:

- if site $k_1 + 1$ and site k_2 are both occupied, then the particle in site k_1 does not move.
- if site k_2 is occupied but site $k_1 + 1$ is empty, then the particle in site k_1 will move into site $k_1 + 1$ with probability dt .
- if site $k_1 + 1$ is occupied but site k_2 is empty then:
 - ★ if site $k_2 - 1$ is empty, the particle in site k_1 will move into site k_2 with probability qdt .
 - ★ if site $k_2 - 1$ is occupied, the particle in site k_1 will move into site k_2 with probability $qpdt$.
- if both site $k_1 + 1$ and site k_2 are empty, then the particle in site k_1 will move into site $k_1 + 1$ with probability $(1 - q)dt$, and:
 - ★ if site $k_2 - 1$ is empty, the particle in site k_1 will move into site k_2 with probability qdt .
 - ★ if site $k_2 - 1$ is occupied, the particle in site k_1 will move into site k_2 with probability $qpdt$.

If site $k_2 - 1$ has a particle, and site k_2 is empty, then the particle will choose the way as following:

- if site k_1 is empty, then the particle in site $k_2 - 1$ will move into site k_2 with probability dt .
- if site k_1 is occupied, then the particle in site $k_2 - 1$ will move into site k_2 with probability $(1 - pq)dt$.

The character of this model is that the particle in site k_1 first choose whether to move through the shortcut and once it has decided to jump through the shortcut, it

will face the problem of whether site $k_2 - 1$ is occupied. If site $k_2 - 1$ is occupied, then the particle which has chosen the shortcut can only move into site k_2 with probability pdt in order to prevent collision. This means that the particle takes a risk to jump the shortcut because once it has chosen the shortcut it can not go back into segment 2, even if site $k_1 + 1$ is empty.

2. Situation 2

If site k_1 has a particle, then the particle will choose the way as following:

- if site $k_1 + 1$ and site k_2 are both occupied, then the particle in site k_1 does not move.
- if site k_2 is occupied but site $k_1 + 1$ is empty, then the particle in site k_1 will move into site $k_1 + 1$ with probability dt .
- if site $k_1 + 1$ is occupied but site k_2 is empty then:
 - ★ if site $k_2 - 1$ is empty, the particle in site k_1 will move into site k_2 with probability qdt .
 - ★ if site $k_2 - 1$ is occupied, the particle in site k_1 will move into site k_2 with probability $qpdt$.
- if both site $k_1 + 1$ and site k_2 are empty, then:
 - ★ if site $k_2 - 1$ is empty, the particle in site k_1 will move into site k_2 with probability qdt , and it will move into site $k_1 + 1$ with probability $(1 - q)dt$.
 - ★ if site $k_2 - 1$ is occupied, the particle in site k_1 will move into site k_2 with probability $qpdt$, and it will move into site $k_1 + 1$ with probability $(1 - qp)dt$.

If site $k_2 - 1$ has a particle, and site k_2 is empty, then the particle will choose the way as following:

- if site k_1 is empty, then the particle in site $k_2 - 1$ will move into site k_2 with probability dt .
- if site k_1 is occupied, then the particle in site $k_2 - 1$ will move into site k_2 with probability $(1 - pq)dt$.

The character of this model is that once the particle has chosen the shortcut but

see that the particle in site $k_2 - 1$ has priority to move into site k_2 , it can change its mind and return to site $k_1 + 1$, providing site $k_1 + 1$ is empty. This model is closer to reality because actually bus drivers can change their mind in time when they are told that the road ahead is to some extent blocked. It is not reasonable for drivers to take a shortcut which is actually difficult to go through.

B. Advanced model 1

Now we divide the lattices into five segments, as shown in figure 2(b), segment 1,2,3,4,5 starts in site 1, $k_1 + 1$, k_2 , $k_3 + 1$, k_4 and ends in site k_1 , $k_2 - 1$, k_3 , $k_4 - 1$, L , respectively.

In this model, all the particles traveling in the lattices are identical to each other. At time $t = 0$, we may suppose that the lattices are empty. As time goes, there will be a particle goes into site 1. This particle will pass continuously through the whole filament.

Given site i ($1 \leq i \leq L$), in an infinitesimal time interval dt , if $i = 1$, a particle is inserted with probability αdt , provided the site is empty, if $i = L$ and there is a particle in it, then the particle in site L is extracted with probability βdt , and if $1 \leq i \leq L$ ($i \neq k_1, k_2 - 1, k_3, k_4 - 1$) and site i is occupied, the particle in site i will move into site $i + 1$ with probability dt , providing site $i + 1$ is empty. However, if a particle is in site k_1 or $k_2 - 1$ or k_3 or $k_4 - 1$, we have to clarify the way it moves. We will construct advanced model 1 using similar method according to basic model (situation 1).

If site k_1 has a particle, then the particle will choose the way as following:

- if site $k_1 + 1$ and site k_2 are both occupied, then the particle in site k_1 does not move.
- if site k_2 is occupied but site $k_1 + 1$ is empty, then the particle in site k_1 will move into site $k_1 + 1$ with probability dt .
- if site $k_1 + 1$ is occupied but site k_2 is empty then:
- ★ if site $k_2 - 1$ is empty, the particle in site k_1 will move into site k_2 with probability

$q_1 dt$.

★ if site $k_2 - 1$ is occupied , the particle in site k_1 will move into site k_2 with probability $q_1 p_1 dt$.

● if both site $k_1 + 1$ and site k_2 are empty, then the particle in site k_1 will move into site $k_1 + 1$ with probability $(1 - q_1) dt$, and:

★ if site $k_2 - 1$ is empty, the particle in site k_1 will move into site k_2 with probability $q_1 dt$.

★ if site $k_2 - 1$ is occupied , the particle in site k_1 will move into site k_2 with probability $q_1 p_1 dt$.

If site $k_2 - 1$ has a particle, and site k_2 is empty, then the particle will choose the way as following:

● if site k_1 is empty, then the particle in site $k_2 - 1$ will move into site k_2 with probability dt .

● if site k_1 is occupied, then the particle in site $k_2 - 1$ will move into site k_2 with probability $(1 - p_1 q_1) dt$.

If site k_3 has a particle, then the particle will choose the way as following:

● if site $k_3 + 1$ and site k_4 are both occupied, then the particle in site k_3 does not move.

● if site k_4 is occupied but site $k_3 + 1$ is empty, then the particle in site k_3 will move into site $k_3 + 1$ with probability dt .

● if site $k_3 + 1$ is occupied but site k_4 is empty then:

★ if site $k_4 - 1$ is empty, the particle in site k_3 will move into site k_4 with probability $q_2 dt$.

★ if site $k_4 - 1$ is occupied , the particle in site k_3 will move into site k_4 with probability $q_2 p_2 dt$.

● if both site $k_3 + 1$ and site k_4 are empty, then the particle in site k_3 will move into site $k_3 + 1$ with probability $(1 - q_2) dt$, and:

★ if site $k_4 - 1$ is empty, the particle in site k_3 will move into site k_4 with probability $q_2 dt$.

★ if site $k_4 - 1$ is occupied , the particle in site k_3 will move into site k_4 with

probability $q_2 p_2 dt$.

If site $k_4 - 1$ has a particle, and site k_4 is empty, then the particle will choose the way as following:

- if site k_3 is empty, then the particle in site $k_4 - 1$ will move into site k_4 with probability dt .
- if site k_3 is occupied, then the particle in site $k_4 - 1$ will move into site k_4 with probability $(1 - p_2 q_2) dt$.

C. Advanced model 2

Also we can divide the lattices into four segments, as shown in figure 3(b), segment 1,2,3,4 starts in site 1, $k_1 + 1$, k_2 , k_3 and ends in site k_1 , $k_2 - 1$, $k_3 - 1$, L , respectively.

In this model, all the particles traveling in the lattices are identical to each other. At time $t = 0$, we may suppose that the lattices are empty. As time goes, there will be particles goes into site 1. These particles will pass continuously through the whole filament.

Given site $i (1 \leq i \leq L)$, in an infinitesimal time interval dt , if $i = 1$, a particle is inserted with probability αdt , provided the site is empty, if $i = L$ and there is a particle in it, then the particle in site L is extracted with probability βdt , and if $1 \leq i \leq L (i \neq k_1, k_2 - 1, k_3 - 1)$ and site i is occupied, the particle in site i will move into site $i + 1$ with probability dt , providing site $i + 1$ is empty. However, if a particle is in site k_1 or $k_2 - 1$ or $k_3 - 1$, we have to clarify the way it moves. We will construct advanced model 2 using similar method according to basic model (situation 1).

If site k_1 has a particle, then the particle will choose the way as following:

- If sites $k_1 + 1, k_2$ and k_3 are all occupied, the the particle in site k_1 does not move.
- If site $k_1 + 1$ is empty but sites k_2 and k_3 are occupied, the particle in site k_1 will move into site $k_1 + 1$ with probability dt .
- If site k_2 is empty but sites $k_1 + 1$ and k_3 are occupied, then:
 - ★ If site $k_2 - 1$ is empty, the particle in site k_1 will move into site k_2 with probability $q_1 dt$.

★ If site $k_2 - 1$ is occupied, the particle in site k_1 will move into site k_2 with probability $q_1 p_1 dt$.

● If site k_3 is empty but sites $k_1 + 1$ and k_2 are occupied, then:

★ If site $k_3 - 1$ is empty, the particle in site k_1 will move into site k_3 with probability $q_2 dt$.

★ If site $k_3 - 1$ is occupied, the particle in site k_1 will move into site k_3 with probability $q_2 p_2 dt$.

● If site $k_1 + 1$ and site k_2 are empty but site k_3 is occupied, then the particle will move into site $k_1 + 1$ with probability $(1 - q_1) dt$, and:

★ If site $k_2 - 1$ is empty, the particle in site k_1 will move into site k_2 with probability $q_1 dt$.

★ If site $k_2 - 1$ is occupied, the particle in site k_1 will move into site k_2 with probability $q_1 p_1 dt$.

● If site $k_1 + 1$ and site k_3 are empty but site k_2 is occupied, then the particle will move into site $k_1 + 1$ with probability $(1 - q_2) dt$, and:

★ If site $k_3 - 1$ is empty, the particle in site k_1 will move into site k_3 with probability $q_2 dt$.

★ If site $k_3 - 1$ is occupied, the particle in site k_1 will move into site k_3 with probability $q_2 p_2 dt$.

● If site k_2 and site k_3 are empty but site $k_1 + 1$ is occupied, then:

★ If site $k_2 - 1$ is empty, the particle in site k_1 will move into site k_2 with probability $q_1 dt$.

★ If site $k_2 - 1$ is occupied, the particle in site k_1 will move into site k_2 with probability $q_1 p_1 dt$.

★ If site $k_3 - 1$ is empty, the particle in site k_1 will move into site k_3 with probability $q_2 dt$.

★ If site $k_3 - 1$ is occupied, the particle in site k_1 will move into site k_3 with probability $q_2 p_2 dt$.

● If sites $k_1 + 1$, k_2 and k_3 are all empty, then the particle in site k_1 will move into site $k_1 + 1$ with probability $(1 - q_1 - q_2) dt$, and:

★ If site $k_2 - 1$ is empty, the particle in site k_1 will move into site k_2 with probability $q_1 dt$.

★ If site $k_2 - 1$ is occupied, the particle in site k_1 will move into site k_2 with probability $q_1 p_1 dt$.

★ If site $k_3 - 1$ is empty, the particle in site k_1 will move into site k_3 with probability $q_2 dt$.

★ If site $k_3 - 1$ is occupied, the particle in site k_1 will move into site k_3 with probability $q_2 p_2 dt$.

If site $k_2 - 1$ has a particle, and site k_2 is empty, then the particle will choose the way as following:

- if site k_1 is empty, then the particle in site $k_2 - 1$ will move into site k_2 with probability dt .

- if site k_1 is occupied, then the particle in site $k_2 - 1$ will move into site k_2 with probability $(1 - p_1 q_1) dt$.

If site $k_3 - 1$ has a particle, and site k_3 is empty, then the particle will choose the way as following:

- if site k_1 is empty, then the particle in site $k_3 - 1$ will move into site k_3 with probability dt .

- if site k_1 is occupied, then the particle in site $k_3 - 1$ will move into site k_3 with probability $(1 - p_2 q_2) dt$.

III. ANALYSIS OF MODELS

A. Theoretical Analysis of the Basic Model

In this section, we will give a theoretical analysis of the phase situation of three segments of our basic model. We choose situation 1 of our basic model described above as the base of our analysis.

According to figure 1(b), we are able to express the insertion and extraction rates

using densities of certain sites. These rates are the following:

$$\begin{aligned}
\alpha_{eff1} &= \alpha \\
\beta_{eff1} &= (1 - \rho_{k_1+1})\rho_{k_2} + (1 - \rho_{k_2})(1 - \rho_{k_1+1})(1 - q) + q(1 - \rho_{k_2})[(1 - \rho_{k_2-1}) + p\rho_{k_2-1}] \\
\alpha_{eff2} &= \rho_{k_1}[\rho_{k_2} + (1 - \rho_{k_2})(1 - q)] \\
\beta_{eff2} &= (1 - \rho_{k_2})[(1 - \rho_{k_1}) + q(1 - p)\rho_{k_1} + (1 - q)\rho_{k_1}] \\
\alpha_{eff3} &= \rho_{k_1}(1 - \rho_{k_2-1})q + \rho_{k_2-1}(1 - \rho_{k_1}) + \rho_{k_1}\rho_{k_2-1} \\
\beta_{eff3} &= \beta \\
J_{sc} &= q\rho_{k_1}(1 - \rho_{k_2})[1 - \rho_{k_2-1} + p\rho_{k_2-1}]
\end{aligned} \tag{4}$$

Certainly, the flux should be conserved:

$$J_1 = J_2 + J_{sc} = J_3 \tag{5}$$

Firstly, we are going to prove that segment 1 and segment 3 should be in the same phase.

If segment 1 is in maximum-current phase, then we have $J_1 = \frac{1}{4}$ according to (3). According to (5), we have $J_3 = \frac{1}{4}$. So segment 3 is also in maximum-current phase.

Secondly, we claim that the following four cases are impossible

$$(L, L, H), (L, H, H), (H, L, L), (H, H, L)$$

To prove that, we firstly focus on the first two situations. If the phase condition is $(L, *, H)$, due to (1), we have:

$$\rho_1 = \alpha \quad \rho_L = 1 - \beta, \quad (\alpha, \beta < 0.5) \tag{6}$$

Because $J_1 = J_3$, we obtain:

$$\alpha(1 - \alpha) = \beta(1 - \beta) \tag{7}$$

When $\alpha \neq \beta$, from (7) we have $\alpha + \beta = 1$, which contradicts $\alpha, \beta < 0.5$. Thus the phase situations $(L, *, H)$ are impossible.

If the phase condition is (H, L, L) , it is difficult for us to analysis the original model with parameter p . So we set $p = 1$, which means that the particle in site k_1 has priority to move into site k_2 when site $k_2 - 1$ is occupied.

Due to (2), we have:

$$\beta_{eff1} = 1 - \rho_{k_1} = 1 - \rho_{k_1+1} + q\rho_{k_1+1}(1 - \rho_{k_2}) \quad (8)$$

from which we can obtain:

$$\rho_{k_1} = \rho_{k_1+1}(1 - q(1 - \rho_{k_2})) \quad (9)$$

Let $1 - q(1 - \rho_{k_2}) = A$, then (9) turns into:

$$\rho_{k_1} = A\rho_{k_1+1} \quad (10)$$

Due to (1), we have:

$$\alpha_{eff2} = \rho_{k_1+1} = \rho_{k_1}[1 - q(1 - \rho_{k_2})] \quad (11)$$

The equation above can be turned into:

$$\rho_{k_1+1} = A\rho_{k_1} \quad (12)$$

From (10) and (12), we reach:

$$\rho_{k_1} = A^2\rho_{k_1} \quad (13)$$

If $\rho_{k_1} = 0$, then $J_1 = \rho_{k_1}(1 - \rho_{k_1}) = 0$ does not agree with reality. So ρ_{k_1} could not be 0. Thus $A = 1$. For all $q \in [0, 1]$,

$$\rho_{k_2} = 1 \quad (14)$$

On the other hand, due to (5), we have:

$$\rho_{k_1}(1 - \rho_{k_1}) = \rho_{k_2}(1 - \rho_{k_2}) \quad (15)$$

Because $\rho_{k_1} = 1 - \beta_{eff1} > 0.5$, $\rho_{k_2} < 0.5$, we know that $\rho_{k_1} \neq \rho_{k_2}$. Together with (15), we have $\rho_{k_1} + \rho_{k_2} = 1$. Together with (14), we get $\rho_{k_1} = 0$. It is impossible. Thus the phase situation can't be (H, L, L) .

If the phase condition is (H, H, L) , also we can set $p = 1$ for our convenience to prove. Because segment 2 is in high-density phase, we have $\beta_{eff2} = 1 - \rho_{k_2-1}$. Because

segment 3 is in low-density phase, we have $\alpha_{eff3} = \rho_{k_2}$. According to (4), we have the cases:

$$\begin{cases} 1 - \rho_{k_2-1} = 1 - \rho_{k_2} - q\rho_{k_1}(1 - \rho_{k_2}) \\ \rho_{k_2} = q\rho_{k_1}(1 - \rho_{k_2-1}) + \rho_{k_2-1} \end{cases} \quad (16)$$

Let $q\rho_{k_1} = B$, then the cases become:

$$\begin{cases} \rho_{k_2-1} = \rho_{k_2} + B(1 - \rho_{k_2}) \\ \rho_{k_2} = \rho_{k_2-1} + B(1 - \rho_{k_2-1}) \end{cases} \quad (17)$$

From (17) we can obtain:

$$(2B - B^2)\rho_{k_2} = (2B - B^2) \quad (18)$$

Because $J_3 = \rho_{k_2}(1 - \rho_{k_2})$, we know that $\rho_{k_2} \neq 1$, together with (18), we will have $B = 0$, which leads to $\rho_{k_1} = 0$, which does not agree with reality. Thus (H, H, L) has been excluded.

Now we have proved that segment 1 and segment 3 should be in the same phase. In the following, we will discuss which phase may segment 2 be in.

If the three segments are in $(L, *, L)$, from the proof which has demonstrated that (H, H, L) is impossible, we know that (L, H, L) can not be true either. Thus we can definitely say that for $(L, *, L)$, only (L, L, L) can be right.

If the three segments are in $(H, *, H)$, let us see whether (H, L, H) is true. If the three segments are in (H, L, H) , then we have:

$$\begin{cases} \beta_{eff1} = 1 - \rho_{k_1} = 1 - \rho_{k_1+1} + q\rho_{k_1+1}(1 - \rho_{k_2}) \\ \alpha_{eff2} = \rho_{k_1}[1 - q(1 - \rho_{k_2})] = \rho_{k_1+1} \end{cases} \quad (19)$$

Let $1 - q(1 - \rho_{k_2}) = A$, then from (19) we can get $\rho_{k_1} = A\rho_{k_1+1}$, then $A = 1$, then $\rho_{k_2} = 1$.

Because of (5), we have:

$$\rho_{k_1}(1 - \rho_{k_1}) = \rho_{k_1+1}(1 - \rho_{k_1+1}) + q\rho_{k_1}(1 - \rho_{k_2}) = \beta(1 - \beta) \quad (20)$$

If we put $\rho_{k_2} = 1$ into (20), we will get:

$$\rho_{k_1}(1 - \rho_{k_1}) = \rho_{k_1+1}(1 - \rho_{k_1+1}) \quad (21)$$

Because $\rho_{k_1} \neq \rho_{k_1+1}$, we have $\rho_{k_1} + \rho_{k_1+1} = 1$. From $\rho_{k_1} = 1 - \beta$, we know that $\rho_{k_1+1} = \beta$, then $J_2 = \beta(1 - \beta) = J_3$, which does not agree to reality. So (H, L, H) is impossible.

Thus we have excluded (H, L, H) . We now know that for $(H, *, H)$, only (H, H, H) is possible.

As for $(M, *, M)$, it is a tough task for us to analyze the situation of segment 2 using equations. But under assistance of numerical simulations, it will be possible for us to find which phase is true for segment 2.

For situation 2 of model A, we can also write the insertion rates and extraction rates at special sites:

$$\begin{aligned} \alpha_{eff1} &= \alpha \\ \beta_{eff1} &= (1 - \rho_{k_1+1}) + q\rho_{k_1+1}(1 - \rho_{k_2})[(1 - \rho_{k_2-1}) + p\rho_{k_2-1}] \\ \alpha_{eff2} &= \rho_{k_1}[\rho_{k_2} + (1 - \rho_{k_2})(1 - q((1 - \rho_{k_2-1}) + p\rho_{k_2-1}))] \\ \beta_{eff2} &= (1 - \rho_{k_2})[(1 - \rho_{k_1}) + q(1 - p)\rho_{k_1} + (1 - q)\rho_{k_1}] \\ \alpha_{eff3} &= \rho_{k_1}(1 - \rho_{k_2-1})q + \rho_{k_2-1}(1 - \rho_{k_1}) + \rho_{k_1}\rho_{k_2-1} \\ \beta_{eff3} &= \beta \end{aligned} \quad (22)$$

The analysis of situation 2 of basic model is the same of the analysis of situation 1, providing $p = 1$, so we do not state it repeatedly.

From the analysis above, we can conclude that for the situation $p = 1$, the three segments should be among the following situations: (L, L, L) , (H, H, H) , $(M, *, M)$. We will do further investigation using numerical simulations in the following sections.

B. Analysis of advanced model 1

In this section, we will give the phase situation of the five segments of advanced model 1. According to figure 2(b), we are able to express the insertion and extraction

rates using densities of certain sites. These rates are the following:

$$\begin{aligned}
\alpha_{eff1} &= \alpha \\
\beta_{eff1} &= (1 - \rho_{k_1+1})\rho_{k_2} + (1 - \rho_{k_2})(1 - \rho_{k_1+1})(1 - q_1) + q_1(1 - \rho_{k_2})[(1 - \rho_{k_2-1}) + p_1\rho_{k_2-1}] \\
\alpha_{eff2} &= \rho_{k_1}[\rho_{k_2} + (1 - \rho_{k_2})(1 - q_1)] \\
\beta_{eff2} &= (1 - \rho_{k_2})[(1 - \rho_{k_1}) + q_1(1 - p_1)\rho_{k_1} + (1 - q_1)\rho_{k_1}] \\
\alpha_{eff3} &= \rho_{k_1}(1 - \rho_{k_2-1})q_1 + \rho_{k_2-1}(1 - \rho_{k_1}) + \rho_{k_1}\rho_{k_2-1} \\
\beta_{eff3} &= (1 - \rho_{k_3+1})\rho_{k_4} + (1 - \rho_{k_4})(1 - \rho_{k_3+1})(1 - q_2) + q_2(1 - \rho_{k_4})[(1 - \rho_{k_4-1}) + p_2\rho_{k_4-1}] \\
\alpha_{eff4} &= \rho_{k_3}[\rho_{k_4} + (1 - \rho_{k_4})(1 - q_2)] \\
\beta_{eff4} &= (1 - \rho_{k_4})[(1 - \rho_{k_4}) + q_2(1 - p_2)\rho_{k_3} + (1 - q_2)\rho_{k_3}] \\
\alpha_{eff5} &= \rho_{k_3}(1 - \rho_{k_4-1})q_2 + \rho_{k_4-1}(1 - \rho_{k_3}) + \rho_{k_3}\rho_{k_4-1} \\
\beta_{eff5} &= \beta \\
J_{sc1} &= q_1\rho_{k_1}(1 - \rho_{k_2})[1 - \rho_{k_2-1} + p_1\rho_{k_2-1}] \\
J_{sc2} &= q_2\rho_{k_3}(1 - \rho_{k_4})[1 - \rho_{k_4-1} + p_2\rho_{k_4-1}]
\end{aligned} \tag{23}$$

Also, the flux should satisfies:

$$J_1 = J_2 + J_{sc1} = J_3 = J_4 + J_{sc2} = J_5 \tag{24}$$

In the following, we will prove that the segment 1,3 and 5 are in the same phase. Conclusions obtained in the proof of basic model will be used in the following steps. For the sake of simplicity, we always set $p_1 = p_2 = 1$.

If one of the segments 1, 3 or 5 is in maximum-current phase, then the other two will also be in maximum-current phase, since $J_1 = J_3 = J_5 = \frac{1}{4}$. Due to (6) and (7), we also can find that the situation is $(L, *, *, *, H)$ is impossible.

In the following, we discuss the situation $(H, *, *, *, L)$. If segment 3 is in low-density phase, then segment 1,2 and 3 will be in $(H, *, L)$. According to the proof of the basic model, we know that this situation is impossible. Because we can regard segment 1,2 and 3 as basic model, only by changing β to be β_{eff3} . This change does not influence the proof since the rate expression of β_{eff3} is independent of the densities of sites in the first three segments.

Similarly, segment 3 can not be in high-density phase since segment 3,4 and 5 will be in $(H, *, L)$. So now we have excluded $(L, *, *, *, H)$ and $(H, *, *, *, L)$.

On the other hand, if the phase situation is $(L, *, *, *, L)$, then we know immediately that segment 3 can not be in high-density phase. So segment 1,3 and 5 should all be in low-density phase. If the situation is $(H, *, *, *, H)$, then we know immediately that segment 3 can not be in low-density phase. So segment 1,3 and 5 should all be in high-density phase.

Since we have proved that (L, H, L) and (H, L, H) does not exist in basic model, so we can conclude that all of the five segments should all be in low-density phase, providing segment 1 is in low-density phase, and all be in high-density phase, when segment 1 is in high-density phase.

As for the situation $(M, *, M, *, M)$, it is difficult for us to analyze it theoretically. So we will use numerical simulations to see whether segment 2 and 4 are in high or low-density phase. The simulations will be presented in the following section.

C. Analysis of advanced model 2

In this section, we are going to demonstrate the phase situation of four segments of advanced model 2. According to figure 3(b), we are able to express the insertion

and extraction rates using densities of certain sites. These rates are stated as follows:

$$\begin{aligned}
\alpha_{eff1} &= \alpha \\
\beta_{eff1} &= (1 - \rho_{k_1+1})\rho_{k_2}\rho_{k_3} + (1 - \rho_{k_2})\rho_{k_1+1}\rho_{k_3}q_1[(1 - \rho_{k_2-1}) + p_1\rho_{k_2-1}] \\
&\quad + (1 - \rho_{k_3})\rho_{k_1+1}\rho_{k_2}q_2[(1 - \rho_{k_3-1}) + p_2\rho_{k_3-1}] \\
&\quad + (1 - \rho_{k_1+1})(1 - \rho_{k_2})\rho_{k_3}[1 - q_1 + q_1[(1 - \rho_{k_2-1}) + p_1\rho_{k_2-1}]] \\
&\quad + (1 - \rho_{k_1+1})(1 - \rho_{k_3})\rho_{k_2}[1 - q_2 + q_2[(1 - \rho_{k_3-1}) + p_2\rho_{k_3-1}]] \\
&\quad + \rho_{k_1+1}(1 - \rho_{k_2})(1 - \rho_{k_3})[q_1[(1 - \rho_{k_2-1}) + p_1\rho_{k_2-1}] + q_2[(1 - \rho_{k_3-1}) + p_2\rho_{k_3-1}]] \\
&\quad + (1 - \rho_{k_1+1})(1 - \rho_{k_2})(1 - \rho_{k_3})[1 - q_1 - q_2 + q_1[(1 - \rho_{k_2-1}) + p_1\rho_{k_2-1}] \\
&\quad + q_2[(1 - \rho_{k_3-1}) + p_2\rho_{k_3-1}]] \\
\alpha_{eff2} &= \rho_{k_1}[\rho_{k_2}\rho_{k_3} + (1 - \rho_{k_2})\rho_{k_3}(1 - q_1) + (1 - \rho_{k_3})\rho_{k_2}(1 - q_2) \\
&\quad + (1 - \rho_{k_2})(1 - \rho_{k_3})(1 - q_1 - q_2)] \\
\beta_{eff2} &= (1 - \rho_{k_2})[(1 - \rho_{k_1}) + q_1(1 - p_1)\rho_{k_1} + (1 - q_1)\rho_{k_1}] \\
\alpha_{eff3} &= \rho_{k_1}(1 - \rho_{k_2-1})q_1 + \rho_{k_2-1}(1 - \rho_{k_1}) + \rho_{k_1}\rho_{k_2-1} \\
\beta_{eff3} &= (1 - \rho_{k_3})[(1 - \rho_{k_1}) + q_2(1 - p_2)\rho_{k_1} + (1 - q_2)\rho_{k_1}] \\
\alpha_{eff4} &= \rho_{k_1}(1 - \rho_{k_3-1})q_2 + \rho_{k_3-1}(1 - \rho_{k_1}) + \rho_{k_1}\rho_{k_3-1} \\
\beta_{eff4} &= \beta \\
J_{sc1} &= q_1\rho_{k_1}(1 - \rho_{k_2})[1 - \rho_{k_2-1} + p_1\rho_{k_2-1}] \\
J_{sc2} &= q_2\rho_{k_1}(1 - \rho_{k_3})[1 - \rho_{k_3-1} + p_2\rho_{k_3-1}]
\end{aligned} \tag{25}$$

Also we have the equation of flux:

$$\begin{aligned}
J_1 &= J_4 = J_3 + J_{sc2} \\
J_3 &= J_2 + J_{sc1}
\end{aligned} \tag{26}$$

Our conclusion is that the segments 1 and 4 are in the same phase. In order to simplify our proof, we always set $p_1 = p_2 = 1$.

Firstly, if segment 1 or segment 4 is in maximum-current phase, then the other should also be in maximum-current phase due to (26).

If the phase condition is $(L, *, *, H)$, then due to $\alpha(1 - \alpha) = \beta(1 - \beta)$, we know that when $\alpha \neq \beta$ we have $\alpha + \beta = 1$, which is contradictory to $\alpha < 0.5$ and $\beta < 0.5$.

Now we discuss the situation $(H, *, *, L)$. If the segment 3 is in low-density phase, then we have:

$$J_1 = \rho_{k_1}(1 - \rho_{k_1}) = J_3 + J_{sc_2} = \rho_{k_3}(1 - \rho_{k_3}) = J_4 \quad (27)$$

and

$$J_3 = \rho_{k_2}(1 - \rho_{k_2}) \quad (28)$$

If segment 2 is in low-density phase, then

$$\alpha_{eff2} = \rho_{k_1} = \rho_{k_1+1}(1 - q_1(1 - \rho_{k_2}) - q_2(1 - \rho_{k_3})) \quad (29)$$

Let $1 - q_1(1 - \rho_{k_2}) - q_2(1 - \rho_{k_3}) = A$, we have:

$$\rho_{k_1} = A\rho_{k_1+1} \quad (30)$$

Since segment 1 is in high-density phase, we have:

$$\beta_{eff1} = 1 - \rho_{k_1} = 1 - \rho_{k_1+1} + q_1\rho_{k_1+1}(1 - \rho_{k_2}) + q_2\rho_{k_1+1}(1 - \rho_{k_3}) \quad (31)$$

So we have:

$$\rho_{k_1+1} = A\rho_{k_1} \quad (32)$$

From (30) and (32), we know that:

$$\rho_{k_1} = A^2\rho_{k_1} \quad (33)$$

From (33) we have $A = 1$ because ρ_{k_1} can not be 0. So $\rho_{k_2} = \rho_{k_3} = 1$ for all $q_1, q_2 \in (0, 1]$, which implies $J_4 = 0$. So segment 2 can not be in low-density phase. If segment 2 is in high-density phase, then we have:

$$\begin{aligned} \beta_{eff2} &= 1 - \rho_{k_2-1} = 1 - \rho_{k_2} - q_1\rho_{k_1}(1 - \rho_{k_2}) \\ \alpha_{eff3} &= \rho_{k_2} = q_1\rho_{k_1}(1 - \rho_{k_2-1}) + \rho_{k_2-1} \end{aligned} \quad (34)$$

From (34) we know that $\rho_{k_1} = 0$ or $\rho_{k_2} = 0$, which will lead to $J_1 = 0$ or $J_3 = 0$ that does not make sense when $q_1 \neq 0$. So we have excluded $(H, *, L, L)$.

If the segment 3 is in high-density phase, then

$$\begin{aligned}\beta_{eff3} &= 1 - \rho_{k_3-1} = 1 - \rho_{k_3} - q_2\rho_{k_1}(1 - \rho_{k_3}) \\ \alpha_{ef4} &= \rho_{k_3} = q_2\rho_{k_1}(1 - \rho_{k_3-1}) + \rho_{k_3-1}\end{aligned}\tag{35}$$

From (35) we know that $\rho_{k_1} = 0$ or $\rho_{k_3} = 0$, which will lead to $J_1 = 0$ or $J_4 = 0$ that does not make sense. $(H, *, H, L)$ is also impossible.

Now we have proved that segment 1 and segment 4 are in the same phase. We now discuss the situation $(L, *, *, L)$. From the proof above we know that segment 3 can not be in high-density phase, either can not segment 2. Thus the phases can only be (L, L, L, L) .

While considering $(H, *, *, H)$, we should first assume that segment 3 be in low-density phase. From the proof above we will know that segment 2 can not be either in low or in high-density phase. And if we assume segment 3 to be in high-density phase, we know that segment 2 can not be in low-density phase. Thus the phases can only be (H, H, H, H) .

To the situation $(M, *, *, M)$, it is rather difficult for us to analyze it theoretically. So we also use numerical simulations to get the results.

IV. NUMERICAL SIMULATIONS OF THE DIFFERENT MODELS

A. Simulations of basic models

In this section, we use numerical methods to simulate basic models. First we demonstrate the results of situation 1 and situation 2.

According to analysis in last section, we set insertion rate α and extraction rate β to be 0.3 and 0.8 for (L, L, L) , 0.8 and 0.3 for (H, H, H) , 0.8 and 0.8 for $(M, *, M)$. The results of the simulations are plotted in figure 4.

From figure 4, we can see that the results of numerical solutions are in agreement with the results of equation analysis. When $\alpha = 0.3$ and $\beta = 0.8$, all the three segments are in low-density phase. When $\alpha = 0.8$ and $\beta = 0.3$, all the three segments

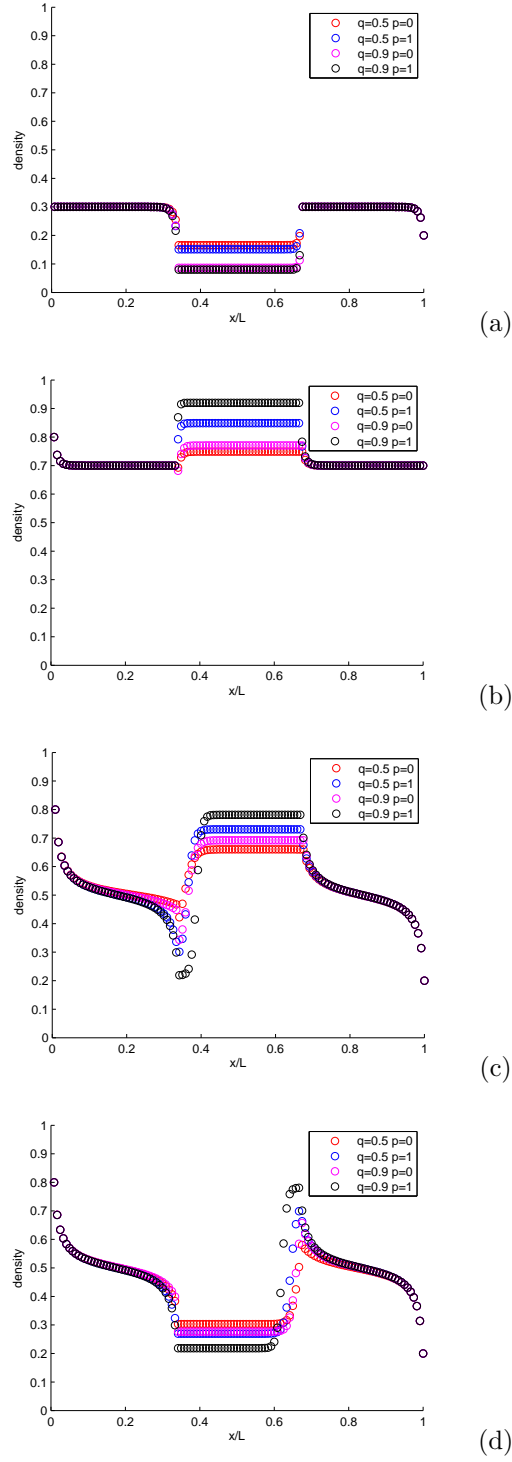


FIG. 4: Numerical simulations of basic model(situation 1) corresponding to different phases:(a) $\alpha = 0.3, \beta = 0.8$; (b) $\alpha = 0.8, \beta = 0.3$; (c) $\alpha = \beta = 0.8$, initial densities of all sites are high; (d) $\alpha = \beta = 0.8$, initial densities of all sites are low.

are in high-density phase. When $\alpha = \beta = 0.8$, segment 1 and segment 3 are in maximum-current phase.

From figure 4(a), we can find that the density in segment 2 decreases when q increases, providing p is constant. For the same q , when p increases, the density in segment 2 decreases.

From figure 4(b), we see that the density in segment 2 increases when q increases, providing p is constant. For the same q , when p increases, the density in segment 2 increases. This is because when p is relatively large, more particles are able to jump through the shortcut to reach site k_2 , thus leaving particles in segment 2 in a serious traffic jam.

From figure 4(c) and (d), we see that when segment 1 and segment 3 are both in maximum-current phase, segment 2 can be either in high-density phase or low-density phase, depending on the initial density rate of each lattice. If the average initial density rate of the lattices is high, then segment 2 will be in high-density phase when the flux becomes stable. if the average initial density rate of the lattices is low, then segment 2 will be in low-density phase when the flux becomes stable.

Now we use numerical methods to simulate situation 2 of basic model. The insertion and extraction rates are set as the same as in the simulation of situation 1.

From figure 5 we are able to see that the results of situation 2 of basic model seem to be similar to the results of situation 1. But after careful inspection, we may find that there are several differences between the results of the two situations, which manifest the distinct characters of these two situations.

In order to do further investigation, we use figures which can show the differences between two situations. We calculate the values which are obtained by subtracting the density of each lattice of situation 2 from the density of each lattice of situation 1. When we do these subtractions, we keep parameters q and p in constant. The results are shown in figure 6. In figure 6(a) and 6(b), we can see that the differences between the density of two situations are within 3×10^{-16} , which means that when $p = 1$, there is nearly no difference between the two situations. That is because when

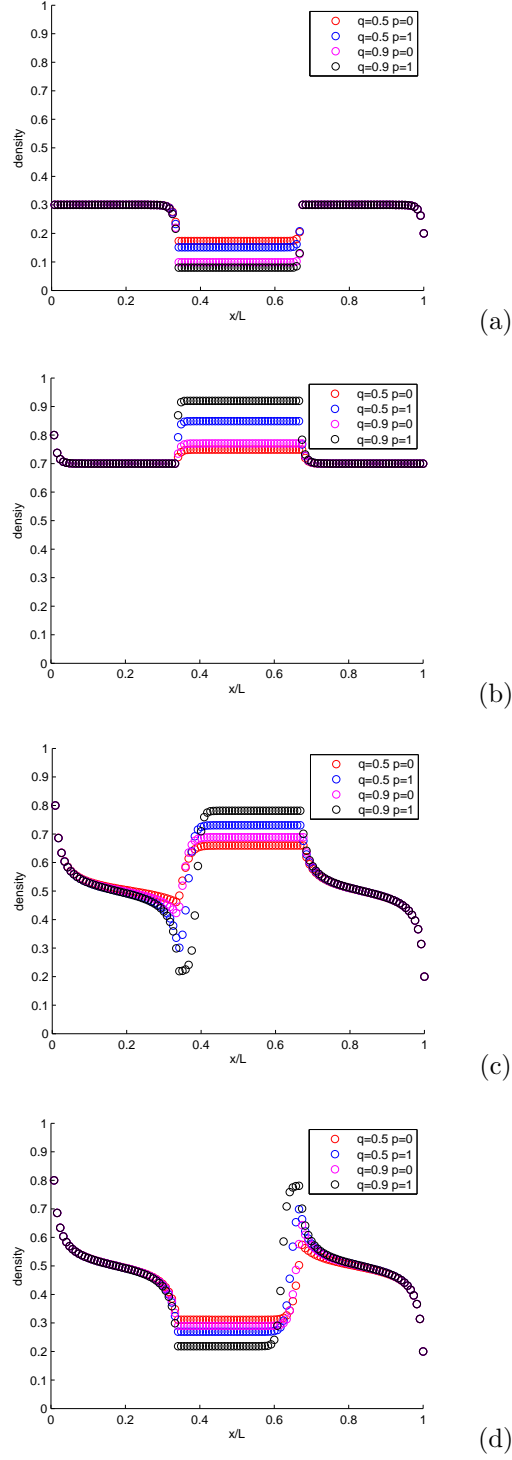


FIG. 5: numerical simulations of basic model(situation 2) corresponding to different phases:(a) $\alpha = 0.3, \beta = 0.8$; (b) $\alpha = 0.8, \beta = 0.3$; (c) $\alpha = \beta = 0.8$, initial densities of all sites are high; (d) $\alpha = \beta = 0.8$, initial densities of all sites are low.

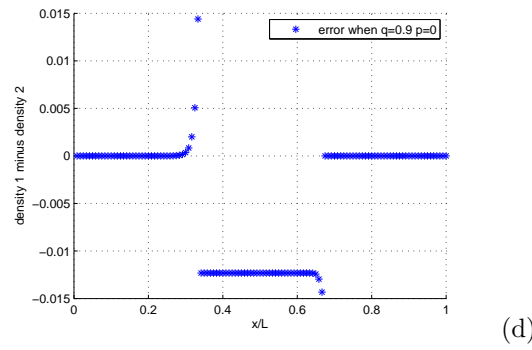
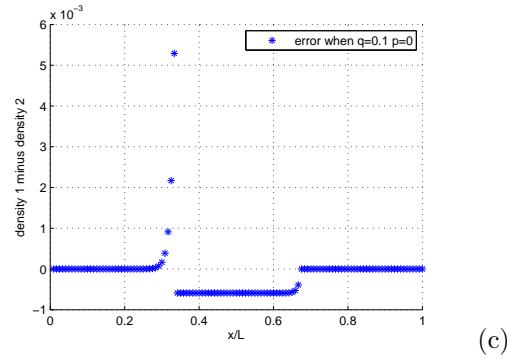
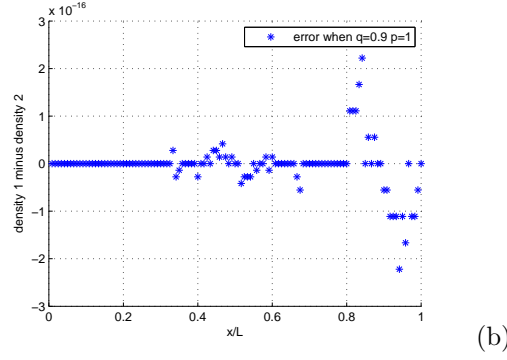
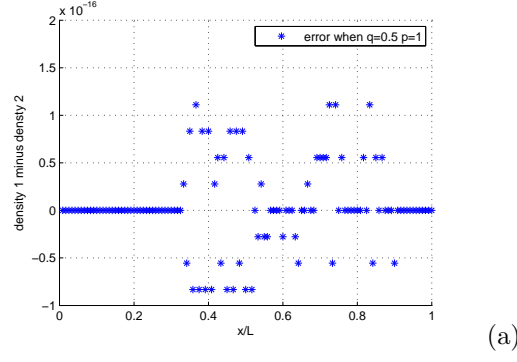


FIG. 6: The density of each lattice of situation 1 of basic model minus the density of each lattice of situation 2 of basic model with insertion rate $\alpha = 0.3$ and extraction rate $\beta = 0.8$: (a) when $q = 0.5$ and $p = 1$; (b) when $q = 0.9$ and $p = 1$; (c) when $q = 0.1$ and $p = 0$; (d) when $q = 0.9$ and $p = 0$.

$p = 1$, the particle at site $k_1(x = \frac{1}{3}$ in the figure) has priority to get to site $k_2(x = \frac{2}{3}$ in the figure). So whatever q may be, the particle can go smoothly once it chooses the shortcut. There is no difference whether it returns back to segment 2 or not.

In figure 6(c) and (d), when $p = 0$, the differences seem to be obvious. when we contrast these two figures, we are able to find that the extent of difference is decided by parameter q . While q changes from 0.1 to 0.9, the error changes from within 6×10^{-3} to 0.015. This is simply because the smaller q means less particles to choose the shortcut. But if we compare figure 6(b) with 6(d), it can apparently be found that the error changes greatly when p is switched from 1 to 0, providing q is comparatively large. So we can say that it really matters whether a particle can return back or not when many particles choose the shortcut. If the particle can not return, a great traffic jam will happen at the certain site, reflected in figure 6(d) as a high-positioned blue star on the top of the figure.

B. Simulations of advanced model 1

Now we use numerical methods to simulate advanced model 1. In order to find how the phases of five segments change when parameters q_1 and q_2 change, we set $p_1 = p_2 = 1$, thus corresponding to our equation analysis in section 3. We will see whether the numerical results correspond to our analytical results.

From figure 7(a), we can see that all of the five segments are in low-density phase. The situation in segment 2 has nothing to do with the situation in segment 4. We can treat them as a connection of two basic models. The density rate in segment 2 decreases when q_1 increases. The density rate in segment 4 decreases when q_2 increases.

From figure 7(b), we can see that all of the five segments are in high-density phase. The situation in segment 2 has nothing to do with the situation in segment 4. We can treat them as a connection of two basic models. The density rate in segment 2 increases when q_1 increases. The density rate in segment 4 increases when q_2 increases.

From figure 7(c) and 7(d), we can see that the situation in segment 2 and 4 are

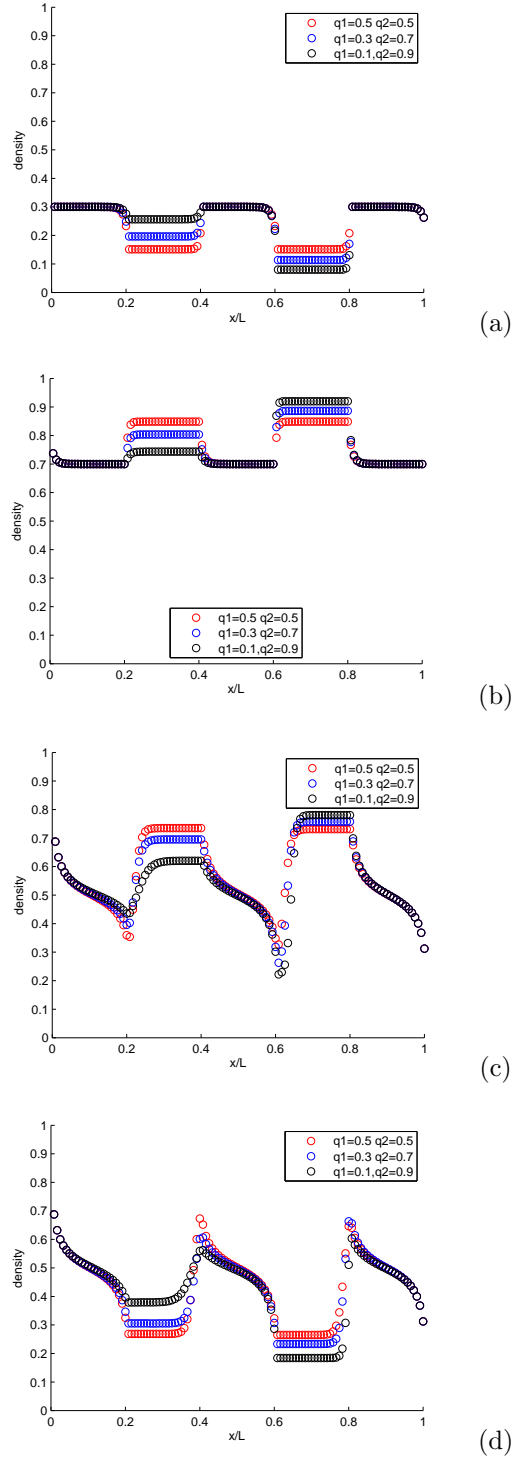


FIG. 7: numerical simulations of advanced model 1 corresponding to different phases: (a) $\alpha = 0.3, \beta = 0.8$; (b) $\alpha = 0.8, \beta = 0.3$; (c) $\alpha = \beta = 0.8$, initial densities of all sites are high; (d) $\alpha = \beta = 0.8$, initial densities of all sites are low.

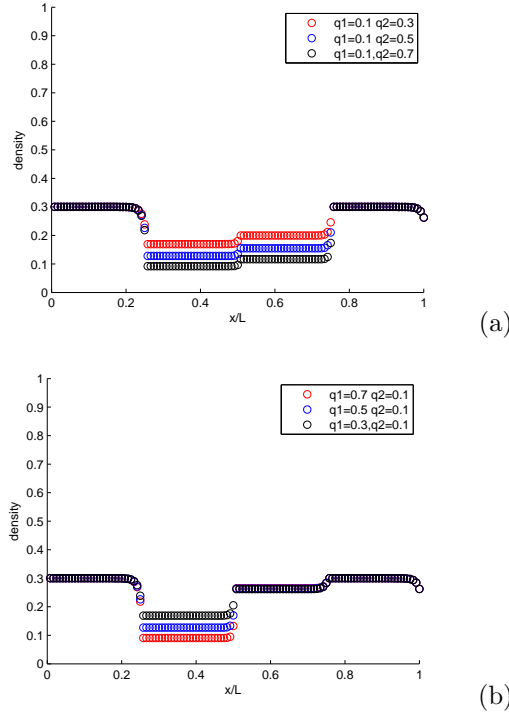


FIG. 8: numerical simulations of advanced model 2 with insertion rate $\alpha = 0.3$ and $\beta = 0.8$: (a) q_2 changes when q_1 remains constant; (b) q_1 changes when q_2 remains constant.

relevant to the initial density of each site. If the average initial density of all sites is high, then segment 2 and 4 will be both in high-density phase. If the average initial density of all sites is low, then segment 2 and 4 will be both in low-density phase.

C. Simulation of advanced model 2

Now we use numerical methods to simulate advanced model 2. In order to find how the phases of four segments change when parameters q_1 and q_2 change, we set $p_1 = p_2 = 1$, thus corresponding to our equation analysis in chapter 3. We will see whether the numerical results correspond to our analytical results. Also, we are interested in whether there are interactions between the two parameters q_1 and q_2 .

First, from figure 8 to figure 11, we see that the numerical simulation results are corresponding to our analytical results. The four segments should be all in low-density phase or high-density phase, and when segment 1 and 4 are in maximum

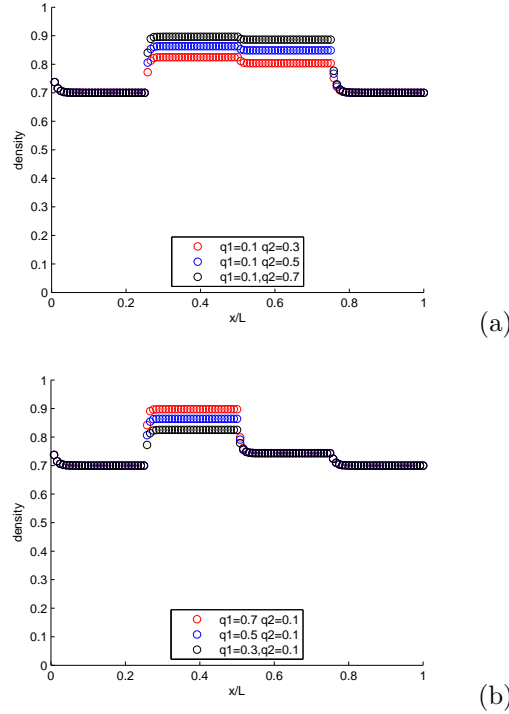


FIG. 9: numerical simulations of advanced model 2 with insertion rate $\alpha = 0.8$ and $\beta = 0.3$: (a) q_2 changes when q_1 remains constant; (b) q_1 changes when q_2 remains constant.

current phase, the phase of segment 2 and 3 will be based upon the initial density of each lattice.

If we focus on figure 8, we will find some interesting things by comparing figure 8(a) and figure 8(b). In figure 8(a), we let q_2 change but q_1 remain constant. We see that both the densities in segment 2 and segment 3 decrease in the same pace when q_2 increases. In figure 8 (b), we let q_1 change when q_2 remains constant. The result is that although the density in segment 2 decreases with increasing of q_1 , the density in segment 3 does not have apparent change. This phenomenon means that the change of q_1 can not influence the density in segment 3 greatly, but the change of q_2 can really influence the density in segment 2 greatly. From this we know that the parameter q_2 has priority to control the density of segment 2 and 3.

From figure 9 to 11, if we compare (a) and (b) of each figure carefully, it is easy for us to find that for each figure, there exists the same phenomenon as in figure 8.

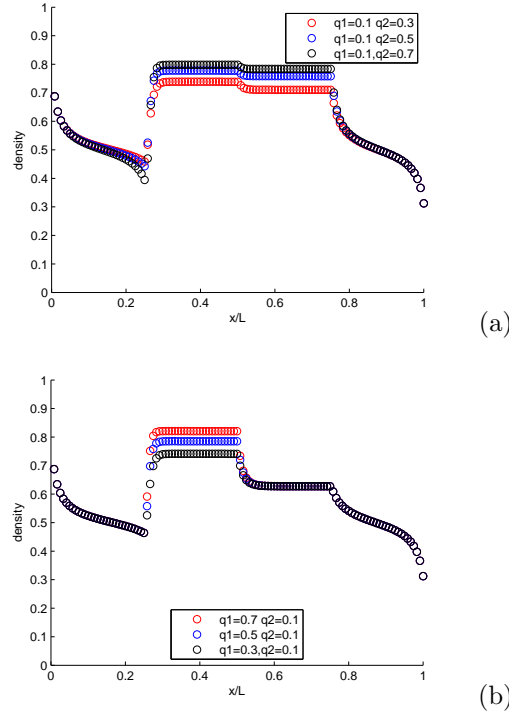


FIG. 10: numerical simulations of advanced model 2 with insertion rate $\alpha = 0.8$ and $\beta = 0.8$ with high initial density for each lattice: (a) q_2 changes when q_1 remains constant; (b) q_1 changes when q_2 remains constant.

In conclusion: in advanced model 2, the density of segment 2 and 3 depends on parameter q_2 greatly. The change of q_2 will bring about the change of density of both of the segments. Once q_2 is set constant, parameter q_1 will also has effects on the density in segment 2, but the density in segment 3 will remain in a relatively stable condition.

V. CONCLUSION

In this paper, TASEP with one or two shortcuts have been analyzed in details. We have found that when a particle arrives at the beginning of the shortcut, it faces the choice of whether to jump through the shortcut or not. If it chooses the shortcut, then a problem exists that whether it can return back to the ordinary road when it finds the road ahead blocked, according to which we have two different situations.

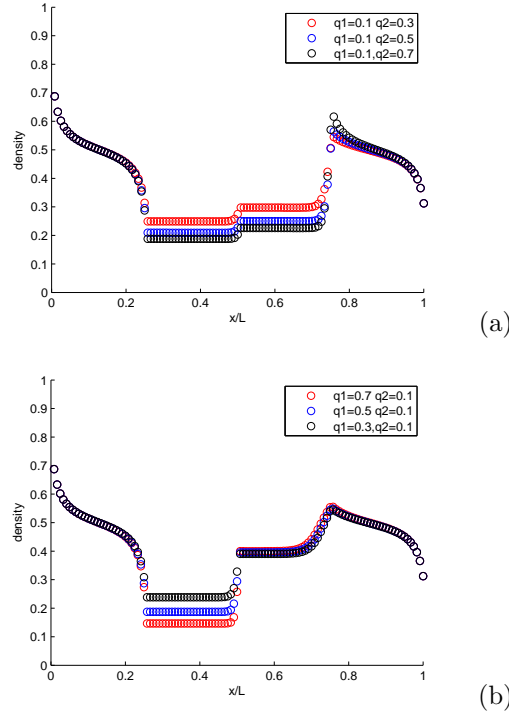


FIG. 11: numerical simulations of advanced model 2 with insertion rate $\alpha = 0.8$ and $\beta = 0.8$ with low initial density for each lattice: (a) q_2 changes when q_1 remains constant; (b) q_1 changes when q_2 remains constant.

After the study we know that if the particle can not return back, the beginning of the shortcut will most probably be filled with particles, producing a heavy traffic jam. This research offers us an idea that when we construct a road with a shortcut, the entrance of the shortcut, that is to say the place where the main road bifurcates, ought to be built widely. Once a driver chooses to jump the shortcut but find the signal showing "blocked ahead", he can immediately turn back to the ordinary road to prevent time-wasting.

From the simulation of basic model, we find that no matter which situation is, the three segments should be in the following four phases:

$$(L, L, L), (H, H, H), (M, L, M), (M, H, M)$$

Whether they are in (M, L, M) or (M, H, M) is based on the initial density of the lattices.

Based on basic model, two advanced models have been studied. One of them have two shortcuts at different places while the other have two shortcuts at the same beginning.

For advanced model 1 we have found that the five segments of the model should be in the following four phases:

$$(L, L, L, L, L), (H, H, H, H, H), (M, H, M, H, M), (M, L, M, L, M)$$

For advanced model 2 we have found that the four segments of the model should be in the following four phases:

$$(L, L, L, L), (H, H, H, H), (M, L, L, M), (M, H, H, M)$$

From the simulation of advanced model 1, we have found that this model can also be regarded as two basic models, the segment 3 of the first being the segment 1 of the second. The probabilities of the choices of a particle facing the two shortcuts do not interfere with each other, which means that the probability of the choice of the particle facing one of the shortcuts does not influence the density in the other segment. This phenomenon leads us to suppose that if there are many shortcuts at different places of the road, we can view them as a connection of basic models.

From the simulation of advanced model 2, we have found that the second shortcut can decide the density of both of the two segments in the middle while the first shortcut can only decide the density in segment 2. It offers us an idea that if we induce more drivers to go directly through the second shortcut, the density of the ordinary road will essentially decrease, providing the whole road is not so crowded.

However, in this paper all of the proofs are under the condition $p = 1$. Future work can be focused on how to proof the phase situations for all $p \in [0, 1]$. Also, two shortcuts overlapping each other will be studied in the future.

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